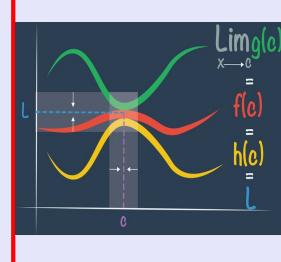


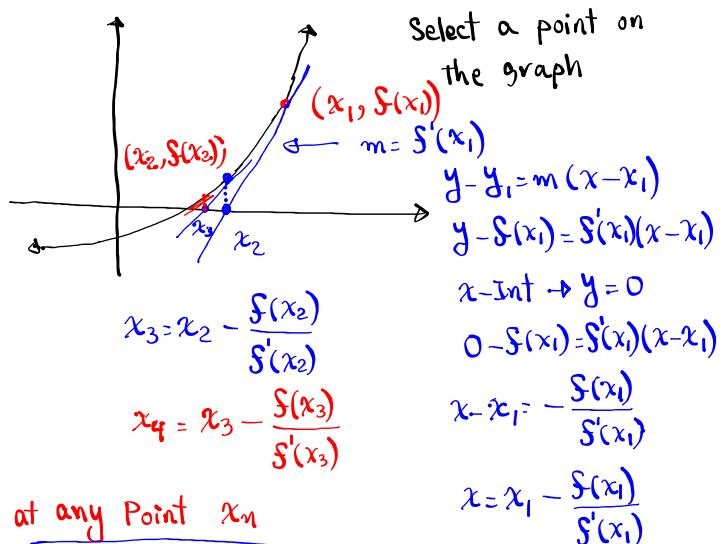
# Math 261

## Spring 2021

### Lecture 33



Consider the function  $y = f(x)$  with graph below



Newton's Method  
to Solve  
 $f(x) = 0$

Solve  $x^3 - 2x - 5 = 0$

$$\text{Let } f(x) = x^3 - 2x - 5 \quad f'(x) = 3x^2 - 2$$

$$\text{Newton's Formula} \quad x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^3 - 2x_n - 5}{3x_n^2 - 2}$$

$$= \frac{x_n(3x_n^2 - 2) - (x_n^3 - 2x_n - 5)}{3x_n^2 - 2} = \frac{2x_n^3 + 5}{3x_n^2 - 2}$$

$$x_{n+1} = \frac{2x_n^3 + 5}{3x_n^2 - 2} \quad \text{Let's start at } x_1 = 1 \quad x_2 = \frac{2(1)^3 + 5}{3(1)^2 - 2} = \frac{7}{1} = 7$$

$$x_3 = \frac{2(7)^3 + 5}{3(7)^2 - 2} = 4.766 \approx 4.8$$

$$x_4 = \frac{2(4.8)^3 + 5}{3(4.8)^2 - 2} \approx 3.4$$

$$x_5 = \frac{2(3.4)^3 + 5}{3(3.4)^2 - 2} \approx 2.6$$

$\vdots$

$$x_{\square} \approx 2.1$$

is an Approximate Soln.

$$f(2.1) = 0$$

$$f(x) = x^3 - 2x - 5$$

$$f(2.1) = (2.1)^3 - 2(2.1) - 5 = .061$$

Use Newton's Method to estimate  $\sqrt[4]{2}$

from calc  $\Rightarrow \sqrt[4]{2} \approx 1.189 \approx 1.2$

we are looking for  $x = \sqrt[4]{2} \quad x^4 = 2$

$$\Rightarrow x^4 - 2 = 0 \quad \text{So let } f(x) = x^4 - 2$$

$$f'(x) = 4x^3$$

Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^4 - 2}{4x_n^3} = \frac{x_n \cdot 4x_n^3 - (x_n^4 - 2)}{4x_n^3} = \frac{3x_n^4 + 2}{4x_n^3}$$

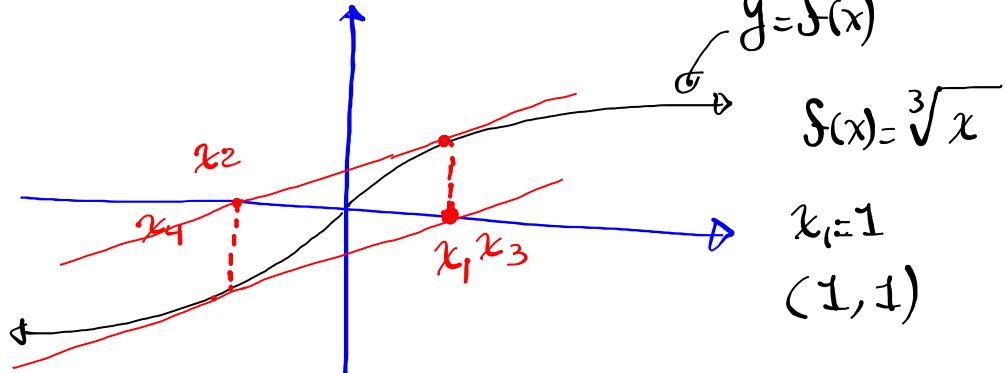
$$x_{n+1} = \frac{4x_n^4 - x_n^4 + 2}{4x_n^3} = \frac{3x_n^4 + 2}{4x_n^3}$$

initial guess at  $x_1 = 1$

$$x_2 = \frac{3 \cdot 1^4 + 2}{4 \cdot 1^3} = 1.25 \quad x_3 = \frac{3 \cdot (1.25)^4 + 2}{4 \cdot (1.25)^3} = 1.1935$$

$$x_4 = \frac{3(1.2)^4 + 2}{4(1.2)^3} = 1.189 \quad \begin{array}{l} \approx 1.2 \\ \text{Newton's Method} \end{array}$$

$\sqrt[4]{2} \approx 1.2$



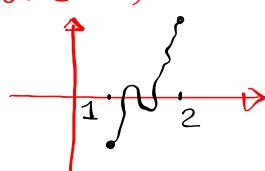
Use Newton's method to Solve  
 $\underbrace{x^4 - 2x^3 + 5x^2 - 6 = 0}_{f(x)} \text{ on } [1, 2]$

$$f(x) = x^4 - 2x^3 + 5x^2 - 6$$

Cont.  $[1, 2]$ , diff.  $(1, 2)$

$$f(1) = 1 - 2 + 5 - 6 = -2$$

$$f(2) = 2^4 - 2(2)^3 + 5(2)^2 - 6 = 14$$



$$f(x) = x^4 - 2x^3 + 5x^2 - 6$$

$$x_1 = 1$$

$$f'(x) = 4x^3 - 6x^2 + 10x$$

$$x_2 = 1.25$$

$$x_{n+1} = x_n - \frac{x_n^4 - 2x_n^3 + 5x_n^2 - 6}{4x_n^3 - 6x_n^2 + 10x_n}$$

$$x_3 = 1.22$$

$$x_{n+1} = \frac{3x_n^4 - 4x_n^3 + 5x_n^2 + 6}{4x_n^3 - 6x_n^2 + 10x_n}$$

$$x_4 = 1.22$$

Soln on  $[1, 2]$

A piece of wire is 10 m long.  
we want to cut this into two pieces.

Make a square from one piece and  
equilateral triangle from the other piece.

How should we cut it so we can have  
max. enclosed area? Minimum enclosed area?

$$4x + 6y = 10$$

$$y = \frac{10 - 4x}{6}$$

$$y = \frac{5 - 2x}{3}$$

**Square:** Side  $x$ , Area  $= x^2$

**Equilateral Triangle:** Side  $2y$ , Area  $= \frac{bh}{2} = \frac{2y \cdot y\sqrt{3}}{2} = y^2\sqrt{3}$

Total Area  $= x^2 + y^2\sqrt{3}$

$$f(x) = x^2 + \left(\frac{5-2x}{3}\right)^2\sqrt{3}$$

$$f(x) = x^2 + \frac{\sqrt{3}}{9}(5-2x)^2$$

$$f'(x)$$

$$f''(x)$$

*Make Sure to finish*

Find the dimensions of the isosceles triangle with largest area that can be inscribed in a circle of radius  $r$ .

